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MODNET
Model Theory and Applications

MVIII.1: Finite Model Theory and Links to Computer Science

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Report on Workpackage MVIII: Finite model theory and links to computer science

In the following, members of the Network are identified by an asterisk ($*$) when first mentioned; external experts and collaborators who were identified as having a close involvement with the project in the original proposal are identified by a double asterisk ($**$).

Preservation theorems for modal logics on special classes of frames were investigated by Dawar** (Cambridge, part of the East Anglia team) and Otto** (Darmstadt, part of the Freiburg team). They showed that the bisimulation-invariant fragment of first-order logic is equivalent to basic modal logic on many natural classes of frames that are of interest in modal correspondence theory, including frames based on pointed equivalence relation, rooted transitive trees, etc. In other cases of interest, such as general rooted frames, or general transitive frames, the bisimulation-invariant fragment is captured by an extension of modal logic with a suitable modality. For example, in the case of rooted frames, it is the extension of modal logic with a universal modality. These results were announced at the LICS conference [1] and are being prepared for journal publication.

These results further develop earlier work of Otto, who presented constructions of finite bisimilar that allow for simplified proofs of the van Benthem-Rosen characterisation theorems and also established a preservation theorem for guarded logic on finite structures in a vocabulary limited to unary and binary relations [2]. The general case of the guarded fragment remains open.

The long-standing open question on the homomorphism preservation theorem on finite structures was settled by Rossman [3], who showed that every first-order sentence preserved under homomorphisms is equivalent, on finite structures, to an existential positive sentence. Independently, Atserias* (Barcelona), Dawar and Kolaitis investigated the status of the preservation property on classes of finite structures more restricted than the class of all finite structures [4]. They showed that the preservation theorem holds on

classes of structures of bounded degree, classes of bounded tree-width and classes that exclude a graph minor. None of these results is implied by Rossman's theorem.

In contrast, the extension preservation theorem, or the Los-Tarski theorem, is known to fail on finite structures. This led to an investigation by Atserias, Dawar and Grohe* (Berlin) of the status of this theorem on "well-behaved" classes of finite structures [5]. They showed that it holds on classes of structures of bounded degree, on acyclic graphs and on the class (for every k) of structures of tree-width at most k. But the preservation property fails on planar graphs and even on planar graphs of tree-width at most 3.

A natural question raised by the cases where such a preservation theorem holds, how much larger is the formula in syntactic normal form, as compared to the original formula of first-order logic with the semantic preservation property. This is being investigated by Dawar, Grohe, Kreutzer* (Berlin) and Schweikardt* (Berlin) who have obtained some non-elementary lower bounds on formula translation in the case of acyclic graphs as well as elementary upper bounds in the bounded-degree case [6].

In the joint work [7] of Makowsky* (Haifa) and Zil'ber* (Oxford) there is an surprising interplay between model theory and finite model theory.

Reference

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[7] J. Makowsky, B. Zil'ber, Polynomial invariants of graphs and totally categorical theories. Preprint 2006.