

MRTN-CT-2004-512234
MODNET
Model Theory and Applications

**MVIII.2: Finite Model Theory and Links to Computer
Science**

Period number: 4 Due date of deliverable: 30/12/08

Period covered: from 1/01/05 to 30/12/08 Date of preparation: 07/02/09

Date of submission: (SESAM)

Start date of project: 1/01/05 Duration: 48 months

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Project coordinator organization name:
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Organization name of lead contractor for this deliverable:
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Project Co-Funded by the European Commission within the Sixth Framework Programme (2002-2006)		
Dissemination in level		
PU	Public	
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Report on Workpackage MVIII: Finite model theory and links to computer science

In the following, members of the Network are identified by an asterisk (*) when first mentioned; external experts and collaborators who were identified as having a close involvement with the project in the original proposal are identified by a triple asterisk (***) .

VIII.1. (a) Modal and guarded fragments, with links to graph and hypergraph theory. (b) Ehrenfeucht-Fraïssé techniques and bisimulation in classical versus modal and guarded logics.

Considerable progress was made by Dawar*** (Cambridge, part of the East Anglia team) and Otto*** (Darmstadt, part of the Freiburg team) [1] with characterisations of modal logics in terms of bisimulation invariance (in the style of classical preservation theorems) over various non-elementary classes of finite frames. These investigations have provided an interesting family of examples of characterisations that partly follow the classical pattern (albeit with entirely different proofs of expressive completeness in the absence of compactness) and partly exhibit specific differences. For instance, over the class of all finite transitive frames, basic modal logic is no longer sufficient for capturing bisimulation invariant first-order definability. Expressive completeness of an extension by a new modality could further be extended to some interesting classes of not necessarily finite transitive frames and to bisimulation invariant monadic second-order definability. Further results related to objectives (a) and (b) in the wider model theory of guarded logics with links to hypergraph theory include work by Blumensath (Darmstadt) [2] and [3] on guarded second-order logic as well as Otto [4] on a new Lindstroem characterisation for guarded logic. Connections between modal logics and other relevant fragments of first-order logic in terms of Ehrenfeucht-Fraïssé techniques in the light of preservation and expressive completeness are also highlighted in the expository survey [5].

VIII 1. (c) Preservation under homomorphisms.

Building on previous work by Atserias*** (Barcelona), Dawar and Grohe* (Berlin) that proved the extension preservation theorem on acyclic finite

structures, David Duris (Paris) studied the property on acyclic hypergraphs [6]. He showed that the extension preservation property is sensitive to the precise notion of acyclicity one considers on hypergraphs. This paper received the Kleene award for the best student paper at LICS 2008, the Twenty-Third Annual IEEE Symposium on Logic in Computer Science (LICS 2008), that was held in Pittsburgh, USA. This is the leading international conference on Logic in Computer Science. The study of the finite model theory of restricted classes of structures was further extended by Dawar and Grohe, along with Kreutzer [7] who introduced classes with “locally excluded minors” and showed that formulas of first-order logic can be efficiently evaluated in such classes. Dawar [8] further extended the scope of the homomorphism preservation theorems established earlier by Atserias, Dawar and Kolaitis by showing that they hold on classes that locally exclude minors and to classes of bounded expansion. That the homomorphism preservation theorem cannot be extended from first-order logic to fixed-point logic was proved by Dawar and Kreutzer [9]. This paper appears in the proceedings volume of ICALP 2008: the International Colloquium on Automata, Languages and Programming. This is the leading European conference in theoretical computer science.

VIII.3: Descriptive complexity and proof complexity, subgoal b, Apply methods from finite model theory, in particular Ehrenfeucht-Fraïssé games, in the context of proof complexity

Atserias, Dalmau (Barcelona) and Dawar in collaboration with Bulatov [10,11,12] developed two related techniques for obtaining instances of constraint satisfaction problems that cannot be shown unsatisfiable by certain wide and well-known classes of algorithmic logic-inspired methods, despite being indeed globally unsatisfiable. Both results provided deeper understanding of the reach of the techniques and suggested new directions in the study of descriptive complexity, particularly the extension of fixed-point logics by means of linear-algebraic operators. This is the subject of a collaboration between Dawar and Grohe and their students [13].

VIII.2: Positive primitive definability and homomorphisms of finite structures, subgoal b: Explore links with universal algebra in applications to constraint satisfaction problems

Dalmau in collaboration with Larose [14] studied algebraic conditions under which constraint satisfaction problems can be solved with very little

resources, namely, in deterministic logspace. He also (partly in collaboration with Carvalho and Krokhn) explored connections between universal algebra and the combinatorial structure of the set of obstructions, i.e, minimal certificates of insatisfiability, of constraint satisfaction problems [15,16].

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