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Model Theory and Applications

MI.4: Pure Model Theory

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Report on Workpackage MI: Pure Model Theory

In the following, members of Modnet teams are identified by an asterisk (*) when first mentioned; Modnet fellows are indicated by a double asterisk (**); external experts and collaborators who were identified as having a close involvement with the project in the original proposal are identified by a triple asterisk (***) .

Results

Task I.1: Theoretical stability and simplicity

Result of Task I.1.e: Find new unstable structures with many stable, stably embedded sets and develop model theory of stable domination; develop \mathcal{o} -minimal and simple analogues, and find connections to thorn forking.

E. Hrushovski*** (Jerusalem) and A. Pillay* (Leeds) gave among other things a systematic treatment of generically stable types and groups, which places ‘stable domination’ and ‘stably dominated groups’ in the right general context [26]. This continues the work on measures begun in [25] by Hrushovski***, Y. Peterzil* (Haifa) and Pillay* in order to prove Pillay’s* conjecture that for a compact group G definable in an \mathcal{o} -minimal theory, the quotient G/G^{00} is a compact Lie group of Lie-dimension equal to the \mathcal{o} -minimal dimension of G .

In mid 2008, discussions began with P. Simon (now Ph.D. student of E. Bouscaren***) among others on generically stable measures in a NIP context. A full account of this theory will be written up by Hrushovski***, Simon and Pillay* in 2009, incorporating aspects of Simon’s Master’s thesis [40]. I. Ben Yaacov* (Lyon) also has a treatment of this material using the continuous logic framework [13, 4].

A. Pillay* proved several related results in 2008, for example the existence of strong germs of definable functions, for generically stable types, and existence of maximal stable quotient for definable groups in NIP theories.

G. Boxall (Ph.D. student of A. Pillay*) has worked on lovely pairs of thorn rank 1 theories, proving superrosiness, and obtaining results on imaginaries.

L. Newelski* (Wrocław) [37] discusses the existence and nature of bounded orbits in the space of types of a group are discussed. He also strengthens some of the results in [25] on the existence of invariant Keisler measures in groups. The new ingredient is a conscious usage of the descriptive set-theoretic complexity of the notions considered.

Result of Task I.1.g: Prove the conjecture of Hrushovski that the existence of a finitely axiomatizable, non-trivial strongly minimal theory is equivalent to the existence of an infinite, finitely presented division ring. Investigate the corresponding conjecture of Ivanov for the trivial case.

No new progress. The paper [14] by T. Blossier* (Lyon) and E. Bouscaren*** mentioned in the period 2 report has now been accepted for publication.

Result of Task I.1.h: Develop stability theory for almost elementary classes, and examine groups in this setting.

The domain of research corresponding to this task was not represented in ModNet at its conception due to the exclusion of external members, and was not taken up during ModNet's existence. Therefore there is no progress to report on this task.

Result of Task I.1.i: Investigate compact abstract theories, with applications to the model theory of Banach spaces.

The domain of research corresponding to this task was not represented in ModNet at its conception due to the exclusion of external members, and became active in ModNet with the hiring of I. Ben Yaacov* (Lyon) in September 2006. At about the same time, the formalism of continuous first order logic came to supersede compact abstract theories as the preferred formalism for investigating Banach spaces as well as other metric structures.

The following progress was achieved by ModNet:

- (1) Study of properties of spaces of random variables. Most importantly, the development of a formalism for the space of random variables with values in a given structure, and theorems stating that if this structure is stable (resp., dependent) then so is the random variable space [4, 5, 6].
- (2) Development of topometric spaces as a formalism for type spaces of metric structures, automorphism groups thereof, and other such objects arising naturally in the model theory of metric structures [7].
- (3) Model theoretic study of classes of Nakano spaces as Banach lattices. Most importantly, proof that in a Nakano space the modular functional is definable in the Banach lattice language, and that atomless Nakano spaces are model complete and stable

(in fact, \aleph_0 -stable and \aleph_0 -categorical up to small perturbations of the exponent function) [8].

- (4) General study of definable groups in stable metric structures [9].
- (5) Theorem: a type-definable group in an \aleph_0 -stable structure is definable [10].
- (6) Theorem: an infinite-dimensional Hilbert space or an atomless probability algebra equipped with a generic automorphism are \aleph_0 -stable up to perturbation of the automorphism [11].
- (7) Study of reflexive representability and stable metrics in Banach spaces and other metric groups [12].

Task I.2: Amalgamation constructions à la Hrushovski

Result of Task I.2.a: Construct omega-stable examples showing n -ampleness gives a proper hierarchy ; build strongly minimal such examples; find connections with pseudo-analytic structures.

Strictly stable examples exhibiting n -ampleness were constructed by Evans* (Norwich) in [21] and connections with Hrushovski's 1-ample strongly minimal sets were established in [22]. However, modifying this construction to produce non-trivial 2-ample ω -stable structures has proved elusive. In related work [23] Ferreira** (Modnet ESR, Norwich) gives a better understanding of the (1-ample) combinatorial geometries of Hrushovski's constructions, proving that the geometries of the (countable, saturated) ternary structures are isomorphic to the geometry of the regular type in the uncollapsed version of the construction. Results of Zilber* (Oxford) and Peatfield*** on pseudo-analytic structures and analytic Zariski structures were discussed in the report MI.2. The paper [42] develops a notion of predimension in the general setting of analytic Zariski structures.

Result of Task I.2.b: Build: nilpotent groups of finite Morley rank and exponent, and class greater than 2; omega-stable Lie algebras over finite fields.

A. Baudisch* (Berlin) has given a frame for the additive collapse [3]. This provides a new proof of the existence of his new uncountably categorical groups. Furthermore it is an essential step for the construction of n -nilpotent groups and Lie-algebras of finite Morley rank with $n > 2$. With this tool the case $n = 3$ is expected to be solved in 2009.

Task I.4: Automorphism groups

Result of Task I.4.a: Examine G -compactness for automorphism groups of countable structures.

R. Pelaez (PhD student of Casanovas*, Barcelona) [39, 16] proved that G -compactness is not preserved under addition of constants. He also gave a new and shorter proof of Newelski's* theorem on the finite diameter of a type-definable Lascar strong type, a general method for interpreting ω -categorical many-sorted theories in ω -categorical one-sorted theories, and hence the existence of non G -compact ω -categorical theories (a previous result of Ivanov* (Wrocław)).

J. Gismatullin** (ESR Freiburg and PhD student of Newelski*) and Newelski* gave a new way to construct a new non- G -compact theory, reducing the problem to constructing a group G where the type-connected component differs from the ∞ -connected component. Also, some new examples of non- G -compact groups are given.

Result of Task I.4.b: Prove that countable saturated, omega-stable structures are reconstructable from their automorphism groups (e.g. via the small index property); prove the small index property for omega-categorical structures from Hrushovski constructions.

Work of Barbina and Macpherson* (Leeds) on the reconstruction problem was reported in Modnet report MI.2, but no progress was made on the small index property for the Hrushovski constructions.

Further results on the remaining tasks not reported so far

Task I.1.d: Develop forking in unstable contexts.

During the last two years theories without the independence property (NIP, also called *dependent* theories) have attracted increasing attention not foreseen in the original proposal, both on a very abstract level (reported below) and in connection with o -minimal questions and problems about stable domination (reported under [I.1.e]).

C. Ealy, K. Krupinski* (Wrocław) and Pillay* have developed the fundamental theory of rosy groups, including \mathfrak{b} -generics, chain conditions, stratified ranks, etc. [20]. They have proved several structural results on groups of small $U^{\mathfrak{b}}$ -rank satisfying NIP and fsg (finitely satisfiable generics), which are a common generalization of results known in the finite Morley rank or the o -minimal case:

- (1) Every superrosy group G with NIP and satisfying hereditarily fsg contains an infinite, definable, abelian subgroup. In particular, if $U^{\mathfrak{b}}(G) = 1$, then G is abelian-by-finite.
- (2) Every rosy group G with NIP, satisfying fsg and having at least one \mathfrak{b} -regular, \mathfrak{b} -generic type is abelian-by-finite. In particular, if $U^{\mathfrak{b}}(G) = \omega^\alpha$, then such a generic type exists, and G is abelian-by-finite.
- (3) Every (rosy) dependent group of $U^{\mathfrak{b}}$ -rank 2 and satisfying hereditarily fsg is solvable-by-finite.
- (4) Each infinite superrosy field whose additive group has fsg is algebraically closed.
- (5) If G has NIP, hereditarily fsg, $U^{\mathfrak{b}}(G) = 2$ and G is not nilpotent-by-finite, then modulo a finite center G is virtually (definably) the semidirect product of the additive and multiplicative groups of an algebraically closed field, and $G = G^{00}$.

They conjecture that an infinite, superrosy field with NIP is either algebraically closed or real closed.

In [31, 34] Krupinski* analyses when an infinite, rosy, NIP group G of finite $U^{\mathfrak{b}}$ -rank interprets an infinite field, generalizing some results known in the finite Morley rank or o-minimal case:

- (1) Let R be a \forall -definable integral domain of positive, finite $U^{\mathfrak{b}}$ -rank. Then the field of fractions F of R is interpretable. Moreover, there is a \forall -definable ring embedding of R onto a subring of F with the same $U^{\mathfrak{b}}$ -rank as F .
- (2) Let A and M be infinite, definable, abelian groups such that A acts faithfully and definably on M as a group of automorphisms, M is A -minimal and $U^{\mathfrak{b}}(M)$ is finite. Then there is an infinite field interpretable.
- (3) Let G be a definable NIP group of finite $U^{\mathfrak{b}}$ -rank which is solvable-by-finite but not nilpotent-by-finite. Then there is an infinite field interpretable in G .
- (4) If K is an infinite, superrosy field with NIP, then for every $n > 0$ we have $K = K^n - K^n$. (This is a little step toward proving the conjecture mentioned above.)
- (5) Let G be a definable NIP group of finite $U^{\mathfrak{b}}$ -rank with hereditarily fsg acting definably on a definable set of $U^{\mathfrak{b}}$ -rank 1. Then, under some general assumption about this action, there is an infinite interpretable field. In particular, if G has a definable subgroup H with $U^{\mathfrak{b}}(G/H) = 1$ such that $G/\bigcap g \in GH^g$ is not solvable by-finite, there is an infinite interpretable field.

Generalizing a result of T. Scanlon that stable fields have no Artin Schreier extension, I. Kaplan** (Lyon), Scanlon and Frank Wagner* (Lyon) have shown that the same is true for NIP fields, and that for fields definable (or even type-definable) in simple theories, there are only finitely many such extension [30].

M. Junker* (Freiburg) and J. Königsmann* (Oxford) examine fields in which model theoretic algebraic closure coincides with relative field theoretic algebraic closure. These are perfect fields with nice model theoretic behaviour. For example, they are exactly the fields in which algebraic independence is an abstract independence relation in the sense of Kim and Pillay*. Classes of examples are perfect PAC fields, model complete large fields and henselian valued fields of characteristic 0.

Newelski* [36] applies the language of topological dynamics to extend the basic notions of stability theory, like generic types in groups, and forking, to general unstable contexts.

H. Adler** (Barcelona, Leeds) has found a new way to make sense of the notion of a ‘1-dimensional’ NIP theory [1]. Motivated by the desire to have a common framework for strongly minimal, o -minimal and C -minimal theories, he defined VC -minimal theories. Using Swiss cheese decompositions he showed that VC -minimal theories are dp -minimal, making it plausible that they allow ‘decomposition’ of types into types of ‘forking-weight 1’. Like o -minimal and C -minimal theories, VC -minimal theories are equipped with a natural topology. It remains to be seen whether forking can be expressed in terms of this topology as Dolich has done in the o -minimal case [19].

Adler** had observed earlier that a theory is NIP if and only non-forking is bounded by a certain function, and that non-forking is bounded at all if and only if it is bounded by a slightly larger function [2]. His question whether NIP is equivalent to boundedness of non-forking was now given a positive answer by Chernikov (PhD student of Baudisch*) and Kaplan** for theories without the tree property of the second kind NTP_2 , which in particular include NIP and simple theories [18]. This generalization of a classical theorem of Harnik and Harrington is the first non-technical result specifically for this large and potentially important class of theories. Chernikov and Kaplan** also answer a question of Pillay*, namely that under NTP_2 forking and dividing over models (and in fact over any extension base) are the same. As a corollary they obtain that dependence is equivalent to bounded forking.

Finally R. Pelaez gave a new example of a non simple non SOP_1 theory.

Task I.3: Topological methods in model theory

Result of Task I.3.a: Develop model theory of the recent notion of a profinite structure, find examples.

Krupinski* extends his preprint [33] by including some topological consequences of the assumption of the existence of m -independent extensions in the home sort (in [33], compact structures with this property are called compact e -structures). Namely, this assumption implies that all orbits are profinite, and that the existence of m -independent extensions also holds over parameters from the home sort together with $\text{acl}^{eq}(\emptyset)$. These results are essential in the generalization of Newelski's* theorems on small profinite structures to the case of compact e -structures. A class of non-trivial examples of compact [profinite] e -structures which are not small is constructed.

In [32], it was proven that each small compact G -group of finite \mathcal{NM} -rank is nilpotent-by-finite, and conjectured that it is abelian-by-finite (even under the weaker assumption that \mathcal{NM} -rank is an ordinal - so called nm -stability). Krupinski* and Wagner* [35] have shown this conjecture in the finite \mathcal{NM} -rank case, using the fact that such groups are nilpotent-by-finite. They have also generalized some other facts proved in [41] for small profinite groups.

A. Ivanov* (Wrocław) studies subgroups G of the (profinite) group of isometries $\text{Iso}(T)$ of a locally finite rooted tree T [27]. Such G is *small* if it only meets a countable number of conjugacy classes of $\text{Iso}(T)$. This notion generalizes that of a profinite group in the sense of Newelski* [38]. In particular, Ivanov studies the question when certain associated zeta functions are rational.

In another direction, Ivanov* considers the profinite completion of a residually finite ω -categorical group. Recall that a subgroup H of an infinite group G is *inert* if it is commensurable with all its G -conjugates. It is observed [28] that any finite subgroup of an ω -categorical group is contained in an infinite residually finite inert subgroup. Ivanov* also defines an embedding of an ω -categorical group G into an infinite symmetric group S such that the closure of G in S is a locally compact group \bar{G} and the closure of any $K < G$ commensurable with H is a compact subgroup of \bar{G} . The model-theoretic properties of \bar{G} are compared to those of G . In particular [28] studies measurable subsets

of \bar{G} . There are some partial results showing that if \bar{G} is small in the sense of Newelski*, then G is very similar to a soluble group.

REFERENCES

- [1] Hans Adler*. Theories controlled by formulas of Vapnik-Chervonenkis codimension 1. Preprint (2008).
- [2] Hans Adler*. Introduction to theories without the independence property. Archive for Mathematical Logic, to appear.
- [3] Andreas Baudisch. The additive collapse. Preprint (2008).
- [4] Itai Ben Yaacov*. Continuous and random Vapnik-Chervonenkis classes. Israel Journal of Mathematics, to appear.
- [5] Itai Ben Yaacov* and H. Jerome Keisler. Randomizations of models as metric structures. Submitted.
- [6] Itai Ben Yaacov*. On theories of random variables. Submitted.
- [7] Itai Ben Yaacov*. Topometric spaces and perturbations of metric structures. Logic and Analysis, volume 1 (2008) no 3-4, 235-272.
- [8] Itai Ben Yaacov*. Modular functionals and perturbations of Nakano spaces. Journal of Logic and Analysis, volume 1 (2009) no 1, 1-42.
- [9] Itai Ben Yaacov*. Stability and stable groups in continuous logic. Submitted.
- [10] Itai Ben Yaacov*. Definability of groups in \aleph_0 -stable metric structures. Submitted.
- [11] Itai Ben Yaacov* and Alexander Berenstein. On perturbations of Hilbert spaces and probability algebras with a generic automorphism. Preprint.
- [12] Itai Ben Yaacov*, Alexander Berenstein and Stefano Ferri. Reflexive representability and stable metrics. Submitted.
- [13] Itai Ben Yaacov*. Transfer of properties between measures and random types. Unpublished research notes.
- [14] Thomas Blossier* and Elisabeth Bouscaren***. Finitely axiomatizable strongly minimal groups. Journal of Symbolic Logic, to appear.
- [15] Gareth Boxall. On lovely pairs of models of a thorn rank 1 theory. Preprint (2008).
- [16] Enrique Casanovas*, Rodrigo Pelaez and Martin Ziegler*. On many-sorted omega-categorical theories. Preprint.
- [17] Enrique Casanovas*. Simple theories, stable formulas and hyperimaginaries. Monograph, to appear.
- [18] Artem Chernikov and Itay Kaplan**. Forking and dividing in NTP_2 theories. Preprint (2008).
- [19] Alfred Dolich. Forking and independence in o-minimal theories. Journal of Symbolic Logic 69:215-240 (2004).
- [20] Clifton Ealy, Krzysztof Krupinski* and Anand Pillay*. Superrosy dependent groups having finitely satisfiable generics. Annals of Pure and Applied Logic, 151:1-21 (2008).
- [21] David Evans*, Ample dividing, J. Symbolic Logic 68 (2003): 1385–1402.
- [22] David Evans*, Trivial stable structures with non-trivial reducts, J. London Math. Soc (2) 72 (2005): 351–363.
- [23] Marco Ferreira**, Geometries of Hrushovski Constructions, PhD Thesis, University of East Anglia, Norwich, March 2009.

- [24] Jacob Gismatullin** and Ludomir Newelski*. G-compactness and groups, Arch.Math.Logic 47:479-501 (2008).
- [25] Ehud Hrushovski***, Ya'acov Peterzil* and Anand Pillay*. Groups, measures, and the NIP. Journal of the American Mathematical Society 21:563–596 (2008).
- [26] Ehud Hrushovski*** and Anand Pillay*. On NIP and invariant measures. Submitted.
- [27] Alexandre Ivanov. Isometry types of profinite groups. Preprint.
- [28] Alexandre Ivanov. \aleph_0 -Categorical groups and their completions. Preprint.
- [29] Markus Junker* and Jochen Königsmann. Schlanke Körper. Submitted.
- [30] Itay Kaplan** and Frank Wagner*. Artin-Schreier extensions of dependent and simple fields. In preparation.
- [31] Krzysztof Krupinski*. Fields interpretable in rosy theories. Israel Journal of Mathematics, to appear.
- [32] Krzysztof Krupinski*. Some model theory of Polish structures. Transactions of the AMS, to appear.
- [33] Krzysztof Krupinski*. Generalizations of small profinite structures. Submitted.
- [34] Krzysztof Krupinski*. Fields interpretable in suprosy groups (a non-solvable case). Submitted.
- [35] Krzysztof Krupinski* and Frank Wagner*. In preparation.
- [36] Ludomir Newelski*. Topological dynamics of definable group actions. Journal of Symbolic Logic, to appear.
- [37] Ludomir Newelski*. Bounded orbits and measures in a group, Israel Journal of Mathematics. Submitted .
- [38] Ludomir Newelski*. Small profinite groups. Journal of Symbolic Logic 66:859–872 (2001).
- [39] Rodrigo Peláez. PhD. Thesis, University of Barcelona, 2008.
- [40] Pierre Simon. Master's Thesis, University of Orsay, 2008.
- [41] Frank Wagner*. Small profinite m-stable groups. Fundamenta Mathematicae 176:181-191 (2003).
- [42] Boris Zilber*, Analytic Zariski structures, predimensions and non-elementary stability, Preprint, July 2008.