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MODNET
Model Theory and Applications

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Dissemination in level		
PU	Public	
PP	Restricted to other programme participants (including the Commission Services)	
RE	Restricted to a group specified by the consortium (including the Commission Services)	
CO	Confidential, only for members of the consortium (including the Commission Services)	

Report on Workpackage MIV: Henselian Fields

In the following, members of Network are identified by an asterisk (*) when first mentioned; MODNET fellows are indicated by a double asterisk (**); external experts and collaborators who were identified as having a close involvement with the project in the original proposal are identified by a triple asterisk (***).

Result of task IV.1.a

Develop geometric model theory for finite extensions of the p-adics (with extra sorts).

Work of Cluckers*** (Leuven), Comte, and Loeser*** (ENS Paris), reported in MIV_3, has now been written up for publication – see [3,4]. The authors study, in several languages of model theory, geometric, metric, and topological aspects of definable p-adic sets and functions, related to geometric integration, Crofton's formula, and Lipschitz continuity.

Bélair and Point* (Mons) [1,2] have studied valued modules, namely modules over a skew polynomial ring and a valuation map from the module to a totally ordered set. Models of these theories of modules are the additive parts of valued fields (of equal or mixed characteristic) with an isometry or a continuous derivation. Under certain conditions on the residue field and value set, they prove a quantifier elimination result and deduce the NIP property for these structures.

Result of task IV.1.c

Study of p-adic integration on definable sets, including the subanalytic case; investigate uniformity issues; connections with the motivic framework.

Work in [6] by Halupczok*** (ENS Ulm) on the description of p-adic definable sets up to isometry is reported in MII_4, under Task II.4.

In [5], previous work by Cluckers is refined by Cluckers()*** and Leeknegt to obtain a p-adic concrete form of resolution of singularities, adapted to the calculation of p-adic integrals.

Result of task IV.2.a

Classify semisimple groups definable in algebraically closed valued fields (ACVF); classify interpretable simple groups; prove cell decomposition in ACVF (possible applications to arc spaces; prove elimination of imaginaries for other important valued structures (e.g. the p-adics or ACVF with subanalytic structure).

Joint work of Hrushovski*** (Jerusalem) and Kazhdan [7] on motivic Poisson summation over function fields has applications to certain semi-simple groups, namely the groups of units of division algebras over such fields (forms of GL_n). The work of Jacquet-Langlands and Deligne-Kazhdan-Vigneras implies highly nontrivial equalities among certain local integrals associated with the Fourier transform; it is shown using motivic Poisson summation that these are valid motivically at the Grothendieck level. The model theory of valued fields plays a role in several ways, in particular in the definition of an integral form of groups of integral points such as $GL_n(R)$ (valuation ring R).

Hrushovski*** and Loeser*** have developed a treatment of Berkovich space in rigid analytic geometry, through the model theory of algebraically closed valued fields (ACVF). If V is a definable set in a model of ACVF, defined over a subfield k with value group contained in $\mathbb{R}$, then the Berkovich space $\mathrm{Berk}(V, k)$ can be identified with the space of types over k which are realised in V and are weakly orthogonal to the value group (a slightly broader class of types than the stably dominated types). They prove a number of results, and a corollary is that all Berkovich spaces of algebraic varieties are of finite topological type. This material has not yet been submitted for publication.

References.

1. L. Bélair, F. Point, Elimination des quantificateurs dans les équations aux différences linéaires sur les vecteurs de Witt, C. R. Acad. Sci. Paris, Ser. I. Math. 346 (2008) 703-706.

2. L. Bélair, F. Point, Quantifier elimination in valued Ore modules, preprint.

3. R. Cluckers, G. Comte, F. Loeser: Lipschitz continuity properties for p-adic semi-algebraic and subanalytic functions, available at http://www.dma.ens.fr/ cluckers/,

4. R. Cluckers, G. Comte, F. Loeser: Local metric properties of p-adic definable sets.

5. R. Cluckers, E. Leenknegt: Rectilinearization of semi-algebraic p-adic sets and Denef's rationality of Poincar series, Journal of Number Theory, Vol. 128, No. 7, 2185–2197 (2008).

6. I. Halupczok, Trees of definable sets over the p-adics. Preprint.

7. E. Hrushovski, D. Kazhdan, Motivic Poisson summation', preprint, arXiv:0902.0845v1