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MODNET
Model Theory and Applications

MIV.2: Progress report on model theory of the p -adics

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Report on Workpackage MIV: Henselian Fields

In the following, members of the Network are identified by an asterisk (*) when first mentioned; external experts and collaborators who were identified as having a close involvement with the project in the original proposal are identified by a double asterisk (**).

Result of task IV.1.a

Develop geometric model theory for finite extensions of the p -adics (with extra sorts).

Cluckers** (Leuven) and Loeser** (Paris VI) [3] have developed a general framework for tame geometry, called b -minimality, which is appropriate for the model theory of all p -adic fields (thus including finite field extensions of the fields of p -adic numbers), and more generally of Henselian valued fields of characteristic zero. Several expansions of such fields are also b -minimal, such as expansions with restricted analytic functions (proved by Cluckers, Lipshitz, Robinson). Hrushovski** (Jerusalem) and Kazhdan have also introduced a notion of tame geometry, called V -minimality (in the ACVF context).

Result of task IV.1.c

Study of p -adic integration on definable sets, including the subanalytic case; investigate uniformity issues; connections with the motivic framework.

Cluckers, Lipshitz [5], and Cluckers, Lipshitz, Robinson [1] have extensively studied the subanalytic case of p -adic integrals and uniformity questions. They have laid the basis for motivic integration in a subanalytic context, and thus motivic integration in any of its meanings can be studied in a subanalytic context by their work on cell decomposition. The notions of b -minimality (Cluckers and Loeser) and of V -minimality (Hrushovski and Kazhdan) generalize the subanalytic case to an abstract setting in which motivic integration is also possible but yet to be explored further. Cluckers and Denef** (Leuven) [8] have studied orbital p -adic integrals.

Result of task IV.1.d

Obtain a p -adic version of the triangulation of bounded definable closed subsets of real affine n -space (simplexes have to be replaced in this p -adic version by another class of simple definable bounded closed sets).

Darniere** (Angers) has obtained a triangulation of p -adic definable sets (preprint not yet available).

Result of task IV.1.e

Study the (restricted) exponential on the field of p -adic numbers (or on various complete extensions of it): in particular, model-theoretic properties and Schanuel's conjecture.

Cluckers and Loeser ([7], [9]) have extensively studied additive characters on p -adic numbers and also uniformity in p , thus motivically. The exponential on the p -adics needs further investigation. Cluckers and Loeser however were able to calculate general p -adic and motivic integrals with this additive character. Cluckers [6] has studied intensively p -adic exponential sums, obtaining partial answers to big open questions by Igusa from the seventies.

Result of task IV.2.a

Classify semisimple groups definable in algebraically closed valued fields (ACVF); classify interpretable simple groups; prove cell decomposition in ACVF (possible applications to arc spaces; prove elimination of imaginaries for other important valued structures (e.g. the p -adics or ACVF with subanalytic structure)).

The monograph [11] was substantially revised, and accepted for publication by the ASL Lecture Notes in Logic (CUP). This concerns development of the theory of stable domination (see also I.1(e)), and of its application to the model theory of algebraically closed valued fields (ACVF). Many results are obtained on ACVF: for example, existence of invariant extensions of types (over an algebraically closed set); identification of stably dominated types with those ‘orthogonal to Γ ’; a proof that ACVF is ‘metastable’ i.e. that, working over a sufficiently rich base, any type is stably dominated over its definable closure in the value group; a maximum modulus principle with an application to groups (characterisation of translation-invariant stably dominated types); identification of non-forking with other forms of independence, over a very nice base; proofs that $\mathbf{C}((t))$ and the theory of differentially closed valued fields are metastable. An interesting invariant of a first order theory

(the semigroup of invariant types, up to domination equivalence) is identified, and investigated for ACVF. There are some connections of this work to Task II.4 (Motivic Integration). Hrushovski has also proved metastability of the rigid analytic expansion of ACVF (without the condition that types have invariant extensions).

In further work, Hrushovski and B. Martin [12] proved elimination of imaginaries for the p -adic field, using results and methods from an earlier part of the above work on ACVF, and thereby proved rationality of a zeta function enumerating (up to an equivalence relation) isomorphism classes of irreducible complex representations of a finitely generated nilpotent group.

N. Guzy* (Mons) [10] identified, uniformly, axioms for the model completion of valued fields of eqcharacteristic 0 equipped with several commuting derivations See also Task II.1(b).

Point* (Mons) and Belair have investigated certain theories of valued difference fields as modules over a skew polynomial ring of the form $K[t, \sigma]$, where (K, v, σ) is a difference valued field where the induced action of σ on the value group G of K is the identity. They axiomatized in that language the theory of the field of Witt vectors $W(E)$, with the Witt Frobenius and the usual valuation, where E is algebraically closed. Then they consider these in a richer language, first adding unary predicates for certain subgroups corresponding to elements of valuation bigger than or equal to a certain value, and obtain a q.e. result modulo index sentences. Finally, they considered three sorted structures (M, w, Δ) of valued modules where M is a R -module, Δ is a totally ordered set endowed with an action of the value group G of K and w is a map from M to Δ satisfying certain axioms. They obtain theories of such structures admitting quantifier elimination, the models of such theories are certain non principal ultraproducts of Witt vectors. Two papers are in preparation.

Result of task IV.2.b

Prove decidability of the field of formal Laurent series over a finite field of prime order.

Major progress has been made towards this by J. Koenigsmann** (associated with Freiburg). In an important piece of work, he has investigated whether, for a perfect ‘almost arbitrary’ field K and function field F over K , a certain group $G_{F/K}$ (which encodes the absolute Galois group of F , with additional information) determines the extension F/K up to isomorphism. He has

proved this for $F = K(t)$. This is expected to shed light on the above question, and also on the undecidability of $\mathbf{C}(t)$, and on a negative solution for Hilbert's 10th Problem over \mathbf{Q} . Several papers are in preparation.

References.

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