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MODNET
Model Theory and Applications

MIV.1: Semisimple groups definable in ACVF

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Dissemination in level		
PU	Public	
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Report on Workpackage MIV: Henselian Fields

In the following, members of Network are identified by an asterisk (*) when first mentioned; external experts and collaborators who were identified as having a close involvement with the project in the original proposal are identified by a double asterisk (**).

Result of task IV.2.a

Task IV.2(a) Classify semisimple groups definable in algebraically closed valued fields (ACVF); classify interpretable simple groups; prove cell decomposition in ACVF (possible applications to arc spaces; prove elimination of imaginaries for other important valued structures (e.g. the p -adics or ACVF with subanalytic structure)).

First, work of Macpherson* (Leeds) and Simonetta* (Paris 7) has now shown that simple groups *definable in the field sort* of an algebraically closed valued field are Chevalley groups over the field. The proof uses earlier work of Prasad on bounded groups, but has not been written up yet. They have some other results in characteristic 0, showing that any definable group has analytic structure.

Very substantial progress has been made by Hrushovski** (Jerusalem) on groups interpretable in ACVF, mainly under the assumption that there is a ‘stably dominated generic type’. Part of this work was completed and written up in 2004, but the manuscript ([3]) is not yet finished. Among many other results are the following.

(1) If G is a generically metastable group definable in the field sort K of ACVF, then there is a definable homomorphism $g : G \rightarrow H(K)$, where H is an algebraic group.

(2) If H is an affine algebraic group, and G a generically metastable definable subgroup of H (in ACVF), then G is isomorphic to $H_1(R)$, where H_1 is an algebraic group scheme over the valuation ring R .

Here *generically metastable* means that the group in question has a stably dominated generic type. This work rests on the development of stable domination in [2]. The latter was revised in 2005.

Haskell, Macpherson, and Mellor, made a serious attempt in May 2005 to prove elimination of imaginaries for structures R' of the following form.

Let R be a polynomially bounded o-minimal expansion of the reals, let R^* be a non-archimedean elementary extension, and let R' be an expansion of R^* be a predicate naming a proper convex valuation ring. Some progress was made, but it does not yet amount to a proof. They will continue to work on the problem.

Progress has been made on cell decomposition theorems in the paper [1]. This handles a range of fields (including ACVF) but with endowed analytic structure (in some cases, handled through separated power series, as developed by Lipshitz and Robinson). A rather uniform cell decomposition theorem (in the sense of Denef) is obtained, and a quantifier elimination result is derived from this. This has analytic motivic applications.

References.

1. R. Cluckers**, L. Lipshitz, Z. Robinson, ‘Analytic cell decomposition and analytic motivic integration’, preprint, math.AG/0503722.
2. D. Haskell, E. Hrushovski, H.D. Macpherson, *Stable domination and independence in algebraically closed valued fields*, monograph submitted to ASL Lecture Notes in Logic.
3. E. Hrushovski, ‘Valued fields, metastable groups’, preprint.