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MODNET
Model Theory and Applications

MI.3: Pure Model Theory

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Report on Workpackage MI: Pure Model Theory

In the following, members of Modnet teams are identified by an asterisk (*) when first mentioned; Modnet fellows are indicated by a double asterisk (**); external experts and collaborators who were identified as having a close involvement with the project in the original proposal are identified by a triple asterisk (***).

Results

Task I.1: Theoretical stability and simplicity

Result of Task I.1.c: Find an omega-categorical, simple, non-low theory.

Recall that a structure is ω-categorical if there are only finitely many pure n-types for all $n < \omega$; it is simple non-low if the local ranks $D(., \varphi(\bar{x}, \bar{y}), k)$ take finite values which tend to infiniy as $k \to \infty$ for at least one φ. In his masters thesis supervised by Frank Wagner* (contractor 4), Vincent Clapiès proved that a simple ω-categorical structure in a finite relationnal with quantifier elimination must be low [1]. Since then there has been no progress on the question, which is currently being studied by Silvia Barbina* (Contractor 12) and Enrique Casanovas* (Contractor 12); the impression is that a possible example should be related to the ω-categorical simple pseudoplane constructed by Hrushovski***.

Result of Task I.3.a: Develop model theory of the recent notion of a profinite structure, find examples.

In [4] Wagner* (Contractor 4) showed that a small profinite group (i.e. profinite group with only countably many orbits under the automorphism group on n-tuples for all $n < \omega$) of ordinal $\mathcal{M}$-rank has finite $\mathcal{M}$-rank and is virtually abelian; he also claimed that for a general small profinite group any orbit has either finite or non well-founded $\mathcal{M}$-rank (the group case of the $\mathcal{M}$-gap conjecture by Ludomir Newelski* (Contractor 10)). However, a gap was pointed out in the proof by Krzysztof Krupinski* (Contractor 10, currently on leave) and Newelski*; this has now been fixed by Wagner* [5].

In his paper [2] Krupinski* generalises the set-up to *compact structures*, pairs (X, G) where X is a compact metric space and G a compact group of homeomorphisms of X. Instead of smallness he assumes that the compact structure satisfies the existence of m-independent extensions (a consequence of smallness, but weaker, as any small compact structure is actually profinite). These appear naturally as spaces of bounded hyperimaginaires (i.e. classes modulo a bounded type-definable equivalence relation) under the action induced by the automorphism group of the ambient structure; in fact any compact structure can be obtained in this way. He shows that just as for small profinite structures, this class is closed under adding imaginaries, and gives an example of a non-small profinite structure with a non-profinite imaginary sort.

As in the profinite case Krupinski* defines $\bar{a}$ to be m-independent of B over A if the orbit $o(\bar{a}/AB)$ is open in $o(\bar{a}/A)$. This satisfies symmetry and transitivity; the $\mathcal{M}$-rank is defined by $\mathcal{M}(a/A) \geq \alpha+1$ if there is $B \not\downarrow^m_A a$ with $\mathcal{M}(a/AB) \geq \alpha$. Assuming existence of m-independent extensions for the structure, or (stronger) for its imaginaries, Krupinski* establishes the Lascar inequalities, the group configuration theorem in the m-normal (one-based) case, existence of regular orbits and the Lascar decomposition of any orbit in an m-stable compact structure into an equidominant finite product of regular orbits.

Even more generally, in [3] Krupinski* develops the notion of a *Polish structure*, a pair (X, G) where G is a Polish group acting (faithfully) on a set X so that the stabilizers of all singletons are closed. This comes with a topological notion of independence, called *nm*-independence (from *non-meager*), and an associated $\mathcal{NM}$-rank, which satisfy the same properties as m-independence and $\mathcal{M}$-rank in profinite structures (in particular, assuming smallness, existence of *nm*-independent extensions and the Lascar inequalities) and which coincides with m-independence in compact structures. Natural examples include the pseudo-arc with the full group of homeomorphisms, moreover this structure is small due to results by Lehner about extending homeomorphisms.

In the case that X is a small Polish G-group, Krupinski* shows that there is a good notion of generic orbit, and the Lascar inequalities for groups hold. Note that contrary to the small profinite case such groups can have elements of infinite order; there even is a torsion-free example of $\mathcal{NM}$-rank 1. Krupinski then analyzes the group-theoretic structure in the compact case: A small compact G-group of ordinal $\mathcal{NM}$-rank is virtually soluble and contains an infinite $*$-closed abelian subgroup; if the $\mathcal{NM}$-rank is finite, the group is virtually nilpotent. He then constructs examples to show that the results fail if the $\mathcal{NM}$-rank is not ordinal.

References

[1] Vincent Clapiès. Simplicité dans les théories ω-catégoriques. Master Thesis, Université Claude Bernard Lyon 1, 2005.

[2] Krzysztof Krupiński. Generalizations of small profinite structures, preprint (2006).

[3] Krzysztof Krupiński. Some model theory of Polish structures, preprint (2007).

[4] Frank O. Wagner. Small profinite m-stable groups. Fundamenta Mathematicæ, 176:181-191, 2003.

[5] Frank O. Wagner. Erratum to "Small profinite m-stable groups", in preparation.