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MODNET
Model Theory and Applications

MI.2: Pure Model Theory

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Report on Workpackage MI: Pure Model Theory

In the following, members of Modnet teams are identified by an asterisk (*) when first mentioned; Modnet fellows are indicated by a double asterisk (**); external experts and collaborators who were identified as having a close involvement with the project in the original proposal are identified by a triple asterisk (***).

Results

Task I.1: Theoretical stability and simplicity

Report on Task I.1.a: Prove a group configuration theorem for 2-simple theories. This was already done and reported in period 1. Recall that Ben Yaacov* (contractor 4), Tomašić (formerly Marie Curie EIF, contractor 4) and Wagner* (contractor 4) had previously shown in [8] an *almost* hyperdefinable group configuration theorem: Given a hyperimaginary tuple $(f_1, f_2, f_3, x_1, x_2, x_3)$ over a hyperimaginary e such that

(1) $f_i \in \mathrm{bdd}(f_j, f_k, e)$,
(2) $x_i \in \mathrm{bdd}(f_j, x_k, e)$, and
(3) all other triples and all pairs are independent over e,

then there is an almost hyperdefinable group. This was improved by Tristram de Piro** (contractor 11), Byunghan Kim and Jessica Young [13], who showed that under the added assumption of model-4-amalgamation one can obtain a group with better definability properties: The underlying domain is hyperdefinable, i.e. the elements are classes modulo a type-definable equivalence relation, rather than merely *almost* hyperdefinable, i.e. classes modulo an invariant equivalence relation boundedly close to a type-definable symmetric relation. The condition of model-4-amalgamation, satisfied in particular in stable theories, is a variant of 2-simplicity as originally defined by Kolesnikov which passes to imaginaries. It is slightly stronger than the more natural 4-amalgamation over models, but weaker than 4-amalgamation over arbitrary sets (which may fail even in stable theories).

Report on Task I.1.b: Isolate conditions under which independence in simple theories is governed by stable formulas, or use (aii) below to obtain counterexamples.

There has been little progress since Byunghan Kim's result [23] that in a supersimple theory, or in a one-based theory with elimination of hyperimaginaries, the canonical base of a Lascar strong type p is the definable closure of the canonical parameters of p-definitions for p-stable formulas φ (i.e. all non-forking extensions have the same φ-type). In [31] Pillay* (contractor 3) shows that a type p of Cantor-Bendixson rank 1 over some countable model $\mathfrak{M}$ is stationary, and asks whether all n-types over $\mathfrak{M}$ are stationary, provided $S_n(\mathfrak{M})$ is countable (i.e. they all have ordinal Cantor-Bendixson rank). If $\mathfrak{M}$ is ω-saturated, the hypothesis implies ω-stability; however, there is a simple unstable example with non-saturated $\mathfrak{M}$ due to Pillay and Kim.

For further progress, it seems that new ideas are needed..

Report on Task I.1.d: Develop forking in unstable contexts (e.g. thorn forking, where one needs to find connections between thorn ranks and ranks in stability/simplicity theory).

In conjunction with his PhD student Wong, Evans* (contractor 1) has looked at ω-categorical Hrushovski constructions and how they relate to Shelah's SOP_n hierarchy [14]. They show that the resulting structures are NSOP_4 and are either simple or have property SOP_3. Also, the natural notion of independence on these structures coming from the predimension coincides with thorn-idependence (a result which also follows from a very general result of Hans Adler** (contractor 12)).

One possible weakening of the stability hypothesis is simplicity. Casanovas* (contractor 12) has reworked and simplified the basic theory [11]; he also gave a course on the subject during the Modnet Summer School at Freiburg. Pillay and Polkowska [28] have generalized work of Hrushovski [19] on PAC bounded substructures of strongly minimal sets to PAC bounded substructures of general stable structures. This yields new examples of properly simple structures, and includes important structures, such as separably closed bounded PAC fields. Wagner* (contractor 4) has studied genericity for arbitrary subgroups of groups interpretable in a simple theory [35], generalizing his earlier work on arbitrary subgroups of stable groups.

Another weakening of the stability assumption concerns theories without the independence property (NIP). Hans Adler** (contractor 12) [1] has shown that a theory is NIP if and only if every type over a model $\mathfrak{M}$ has at most $2^{|T|+|\mathfrak{M}|}$ non-forking global extensions. Since a maximal bound, if such exists, would be $2^{2^{|T|+|\mathfrak{M}|}}$, he asks whether the existence of a bound already implies NIP. Pillay [30] has given a simplified proof of Shelah's theorem of quantifier elimination for NIP structures enriched by the traces of sets definable in an elementary extension.

Groups with NIP have been studied by Hrushovski***, Peterzil* (contractor 9) and Pillay [18]: They have a unique type-definable normal subgroup of bounded index, and abelian subgroups are contained in definable abelian subgroups. If there are finitely satisfiable generic types, there exists an invariant Keisler measure on definable sets; moreover a NIP field with finitely satisfiable generics is algebraically closed.

Pillay [29] has studied canonical bases in rosy theories (where thorn forking yields a rudimentary notion of independence), and derived a condition for their existence.

Finally, in a series of papers [24, 25] Ludomir Newelski* (contractor 10) and Marcin Petrykowski** (contractor 4) have studied a generalisation of the notion of a generic type of a group, a *weak* generic type, which exists in any group, and used this notion to show that if a suitably saturated group is covered by countably many type-definable sets, then the cube of one of the sets is left generic; if the group is amenable, a square suffices.

Report on Task I.1.e: Find new unstable structures with many stable, stably embedded sets and develop model theory of stable domination.

The monograph *Stable domination and independence in algebraically closed valued fields* [16] by Deirdre Haskell, Ehud Hrushovski and Dugald McPherson* (contractor 3) has been revised and accepted by CUP for the ASL Lecture Notes in Logic. It develops the general theory of stable domination (basics, results on extending and restricting the base, strong codes for germs), and discusses the meaning of stable domination in algebraically closed valued fields. Key results have appeared in [15].

Intermediate report

Report on Task I.1.e: Develop o-minimal and simple analogues, and find connections to thorn forking.

In [18] already mentioned above, Hrushovski, Peterzil and Pillay define the notion of *compact domination*. They relate compact domination to the existence, uniqueness and smoothness of suitable Keisler measures, and prove that in the cases we understand well, definable compact groups in *o*-minimal structures are compactly dominated. They conjecture that any definably compact group (definable in a saturated *o*-minimal expansion of a real closed field) is compactly dominated by the compact Lie group G/G^{00}.

Report on Task I.1.g: Prove the conjecture of Hrushovski that the existence of a finitely axiomatizable, non-trivial strongly minimal theory is equivalent to the existence of an infinite, finitely presented division ring. Investigate the corresponding conjecture of Ivanov for the trivial case.

Thomas Blossier* (contractor 4) and Elisabeth Bouscaren*** (Orsay, formerly contractor 2) [9] have shown that if G is a finitely axiomatizable strongly minimal group, then its skew field of quasi-endomorphisms has finite presentation as a ring.

Task I.2: Amalgamation constructions à la Hrushovski

Report on Task I.2.a: Construct omega-stable examples showing n-ampleness gives a proper hierarchy; build strongly minimal such examples; find connections with pseudo-analytic structures.

New ideas seem to be needed in order to find ω-stable examples for the n-ampleness hierarchy.

As for the connection with pseudo-analytic structures, a tentative definition of a pseudo-analytic Zariski structure was given by Peatfield and Zilber* (contractor 5) in 2005 [27]; they also showed that one of Hrushovski's examples (in its uncollapsed, hence infinite rank, incarnation) fits into this framework. More recently, Peatfield [26] has constructed an analytic Zariski structure extending an algebraically closed field, using Hrushovski's amalgamation method.

In a more general vein, Baudisch* (contractor 14), Blossier, Hasson* (contractor 5), Hils* (contractor 14), Martin Pizarro* (contractor 4), Wagner and Ziegler* (contractor 6) have studied the fusion of two striongly minimal sets over a common ω-categorical reduct, and expansions of algebraically closed fields by a predicate for a proper connected additive or multiplicative subgroup [3, 4, 5, 6, 7, 10, 17]. This is based on work of Hrushovski [18] on the fusion and Poizat* (contractor 4) on expansions of algebraically closed fields [32, 33].

Finally, Pourmahdian and Wagner* [34] have constructed a simple positive Robinson theory where Lascar strong type is different from strong type (another example had previously been obtained by Hrushoski). However, the question remains open for first-order theories.

Task I.4: Automorphism groups

Report on Task I.4.a: Examine G-compactness for automorphism groups of countable structures.

Ivanov* (contractor 10) [21] studies connections between G-compactness and the existence of strongly determined types (called *invarinat* types in [16]). In particular, he shows that admitting strongly determined 1-types (i.e. each 1-type extends to a strongly determined type) and type-definability of equality of Lascar strong type together imply that strong types are Lascar strong. Moreover, in this case the theory admits strongly determined n-types over finite sets for all n (a very strong version of G-compactness). This can be applied to show that every weakly o-minimal theory admits strongly determined types, and that each C-minimal theory with type-definable equality of Lascar strong type admits strongly determined types.

In a second paper [22], Ivanov constructs an ω-categorical theory which is not G-compact and which is not geometrically finite (in the sense of Cherlin and Hrushovski [12]).

Report on Task I.4.b: Prove that countable saturated, omega-stable structures are reconstructable from their automorphism groups (e.g. via the small index property); prove the small index property for omega-categorical structures from Hrushovski constructions.

Barbina* (contractor 12) and McPherson show [1], using Rubin's reconstruction technique and results on comeagre conjugacy classes, that for certain homogeneous structures $\mathfrak{M}$, any other ω-categorical structure with abstractly isomorphic automorphism group is bi-interpretable with $\mathfrak{M}$.

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